

Name:

Date:

Period: 6

Trapezoids and Kites

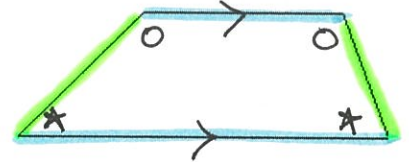
Lesson Objectives

INBAT use properties of trapezoids and kites; identify and name quadrilaterals

Trapezoid (DEF): A trapezoid is a quadrilateral with exactly one pair of parallel sides.

→ The parallel sides are called the bases

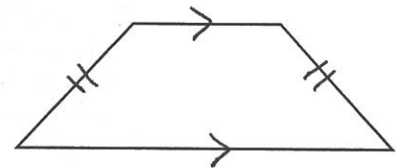
→ The non-parallel sides are called the legs



Base Angles: The base angles of a trapezoid are two consecutive angles whose common side is a base. *o

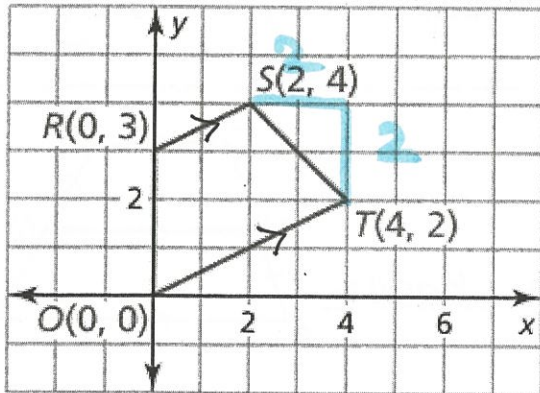
→ A trapezoid always has 2 pairs of base angles

Isosceles Trapezoid: If the legs of the trapezoid are $\cong$, then the trapezoid is isosceles.



Example:

1. Show that ORST is a trapezoid. Then decide whether it is an isosceles trapezoid.



$$\text{slope } \overline{RS} = \frac{1}{2}$$

$$\text{slope } \overline{ST} = -1$$

$$\text{slope } \overline{OT} = \frac{2}{4} = \frac{1}{2}$$

$$OR = 3$$

$$ST^2 = 2^2 + 2^2$$

$$ST = 2\sqrt{2}$$

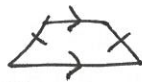
$$\text{slope } \overline{OR} = \text{undef.}$$

Since the opp. sides $\overline{RS}$ and $\overline{OT}$ of quad. ORST have $=$ slopes, then $\overline{RS} \parallel \overline{OT}$. Since the other pair of opp. sides in quad. ORS have unequal slopes, then $\overline{OR}$ is not $\parallel$ to $\overline{ST}$. Therefore, quad. ORS is a trapezoid since only one pair of opp. sides is $\parallel$. Since the legs of trap. ORST are not $=$, then the legs are not $\cong$, and thus trap. ORST is not isosceles.

Isosceles Trapezoid Base Angles Theorem

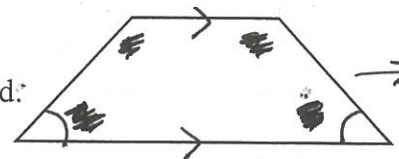
If a trapezoid is isosceles, then each pair of base angles is congruent.

if



Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid:



→ isos trapezoid

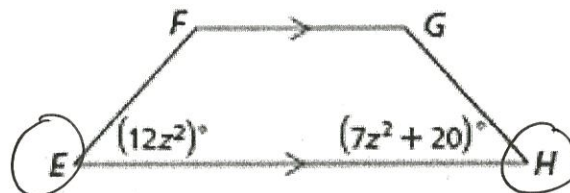
Example:

2. Find the value of z so that trapezoid EFGH is isosceles.

$$12z^2 = 7z^2 + 20$$

$$5z^2 = 20$$

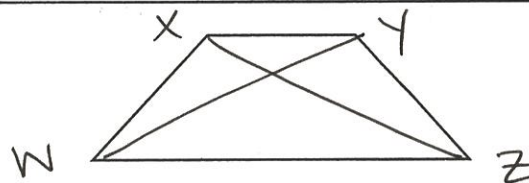
$$z^2 = 4 \rightarrow \boxed{z = \pm 2}$$



Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

$$\overline{WY} \cong \overline{XZ}$$



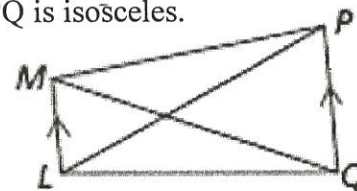
Example:

3. $MQ = 7y - 6$ and $LP = 4y + 12$. Find the value of y so that trapezoid LMPQ is isosceles.

$$7y - 6 = 4y + 12$$

$$3y = 18$$

$$\boxed{y = 6}$$



REVIEW: A midsegment of a **triangle** is the segment that connects 2 midpoints of sides.

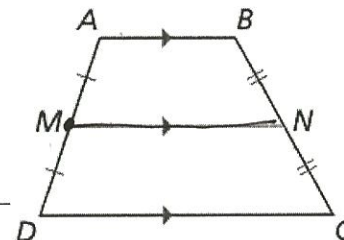
The midsegment of a **trapezoid** is the segment that connects 2 midpoints of legs.

Trapezoid Midsegment Theorem

The midsegment of a trapezoid is...

1. // to each base
2. $\frac{1}{2}$ the length of the sum of the lengths of the bases

$$\overline{MN} = \frac{AB + CD}{2}$$

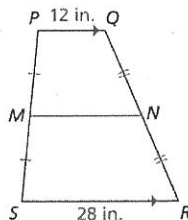


Example:

4. In the diagram, $\overline{MN}$ is the midsegment of trapezoid PQRS. Find MN.

$$MN = \frac{12 + 28}{2}$$

$$= \frac{40}{2} = \boxed{20}$$

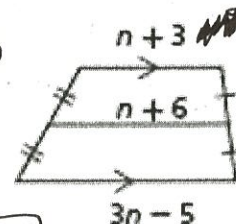


5. Find the value of n .

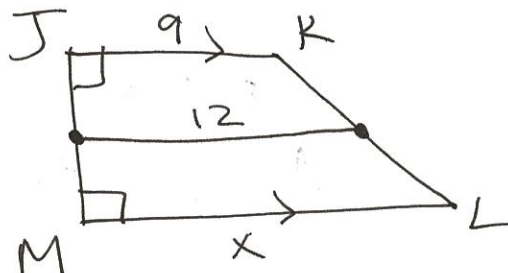
$$n + 6 = \frac{(n + 3) + (3n - 5)}{2}$$

$$2(n + 6) = \frac{4n - 2}{2} \cdot 2$$

$$2n + 12 = 4n - 2 \rightarrow \boxed{n = 7}$$



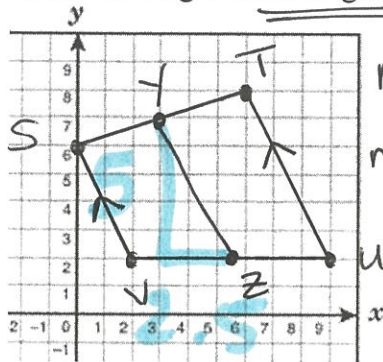
6. In trapezoid JKLM, $\angle J$ and $\angle M$ are right angles, and $JK = 9$ cm. The length of midsegment $\overline{NP}$ of the trapezoid is 12 cm. Sketch the trapezoid and its midsegment, and find the length of $\overline{ML}$.



$$12 = \frac{9 + x}{2}$$

$$x = 15 \rightarrow \boxed{ML = 15}$$

7. Find the length of midsegment YZ in trapezoid STUV.



$$\text{mdpt } \overline{ST} = (3, 7)$$

$$\text{mdpt } \overline{UV} = (5.5, 2)$$

$$YZ^2 = 5^2 + 2.5^2$$

$$YZ^2 = 5^2 + \left(\frac{5}{2}\right)^2$$

$$YZ^2 = 25 + \frac{25}{4}$$

$$YZ^2 = \frac{100}{4} + \frac{25}{4}$$

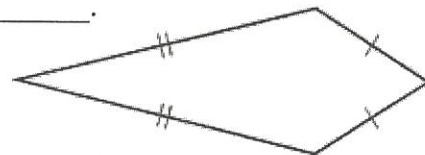
$$\sqrt{YZ^2} = \sqrt{\frac{125}{4}}$$

$$YZ = \frac{\sqrt{125}}{\sqrt{4}} = \sqrt{5} \cdot 2.5$$

$$YZ = \frac{5\sqrt{5}}{2}$$

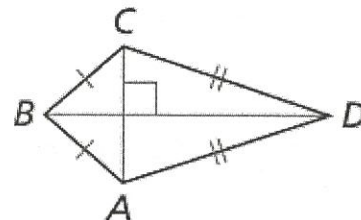
Kite (DEF): A kite is a quadrilateral that has two pairs of _____ sides,

BUT _____ sides are _____.



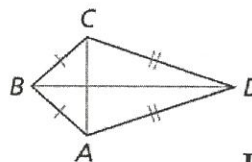
Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.



PROOF: **Given:** ABCD is a kite

Prove: $\overline{AC} \perp \overline{BD}$

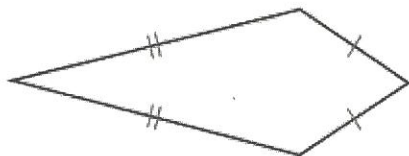


Statements

Reasons

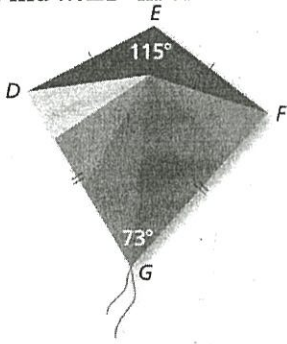
Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.



Example

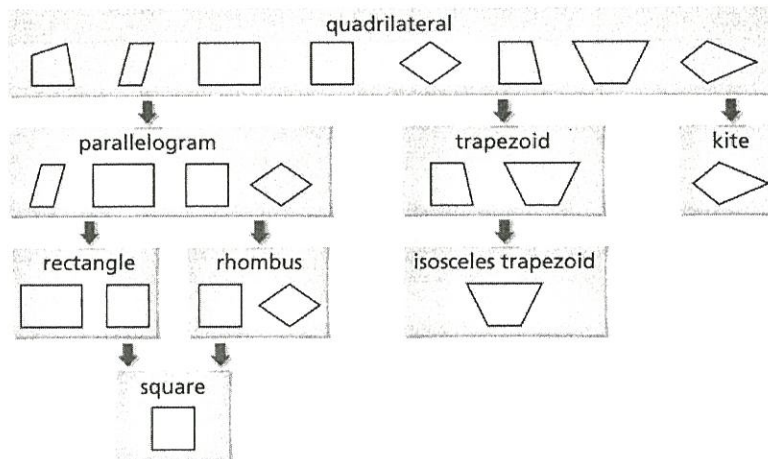
8. Find $m\angle D$ in the kite shown.



9. In a kite, the measures of the angles are $3x^\circ$, 75° , 90° , and 120° . Find the value of x . What are the measures of the angles that are congruent?

Identifying a Quadrilateral

The following diagram shows the relationships among the special quadrilaterals you studied in the chapter.



Example:

10. Give the most specific name for the quadrilateral. Briefly explain your reasoning.

